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The dynamics of inflation and currency substitution in a small open economy

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Abstract

This paper analyzes the relationship between money and inflation in a small open economy, where domestic and foreign currencies are perfect substitutes as means of payment. It is shown that, if the path of domestic money supply is such that individuals find it optimal to change the currency in which transactions are settled, there will be an adjustment period during which domestic inflation adjusts to equal the foreign inflation rate. The model captures the stylized fact that temporary increases in the inflation rate may have permanent effects in the use of foreign currency, even without the introduction of dollarization costs.
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1. Introduction

In high inflation countries, the rapid erosion of the value of domestic currency leads agents to substitute it with foreign currency in any or all of the basic functions of money. This phenomenon is usually called currency substitution, or simply dollarization (for a literature review, see Giovannini and Turtelboom, 1994). A question that has attracted increasing attention is that, in some cases, dollarization exhibits *hysteresis*, with the demand for foreign currency remaining high after stabilization (for empirical evidence, see Guidotti and Rodriguez, 1992; Kamin and Ericsson, 2003; Clements and Schwartz, 1993; Mueller, 1994; Reding and Morales, 1999). However, this is not always the case. In some economies of Eastern Europe,

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for example, dollarization has fallen substantially in the aftermath of successful stabilization plans (Sahay and Végh, 1996).

This paper follows Guidotti and Rodriguez (1992); Uribe (1997), and Reding and Morales (1999), addressing the issue of dollarization hysteresis with a small open economy model where domestic and foreign currencies are perfect substitutes as means of payment. Our approach contributes to the literature in two main respects. Firstly, we allow the inflation rate to be endogenous. Secondly, we are able to explain the phenomenon of dollarization *hysteresis* even in the absence of dollarization costs.

Previous explanations for the phenomenon of dollarization hysteresis have emphasized the role of dollarization costs. Dornbusch and Reynoso (1989) and Dornbusch et al. (1990) argued that the process of financial adaptation involves learning costs that, once incurred by economic agents, imply persistence. Guidotti and Rodriguez (1992) present a model with perfect means of payment substitutability, in which agents face costs of adjusting their holdings of foreign currency. These costs result in an inflation band within which agents choose not to switch between currencies. Hence, de-dollarization may only be achieved if domestic inflation decreases enough to offset the switching costs. Perfect means of payment substitutability was also assumed by Uribe (1997) and Reding and Morales (1999), who stressed the role of network externalities as the source of non-linearities in the relationship between money demand and inflation. According to this interpretation, transaction costs faced by individual agents decline with the aggregate level of dollarization, giving rise to multiple equilibria and history dependence in the demand for money. Thus, a temporary high level of inflation can start a dollarization process which will not necessarily be reversed when inflation comes down, as network economies will provide agents with a permanent lowering of transaction costs in the use of foreign currency.

Guidotti and Rodriguez (1992); Uribe (1997), and Reding and Morales (1999) have two important shortcomings. First, the inflation rate is assumed to be exogenous. As changes in the money demand impact on the inflation rate, the dynamic analysis made by these authors may be misleading. Second, these approaches involve the specification of dollarization cost functions, which are necessarily ad-hoc.

In this paper we argue that the short-term dynamics of money and inflation under the assumption of perfect means of payment substitutability is sufficiently rich to capture the different patterns of dollarization identified in the empirical literature, without the need to introduce dollarization costs. A well known result with perfect substitutability is that, without binding constraints on currency holdings, the exchange rate is undetermined (Kareken and Wallace, 1981). In short, any monetary equilibrium with two currencies coexisting in the same commodity domain requires their user costs (inflation rates) to be equal. Under these circumstances, the demand for each currency is undetermined and so, too, will be the (unchanging) exchange rate. The same mechanism that produces exchange rate indeterminacy in Kareken and Wallace (1981) is at work here, but our model differs in that residents of both countries face binding constraints in the use of currencies. In particular, we assume that foreign residents are not allowed to hold the

domestic currency (only *asymmetric* currency substitution is considered) and a minimum constraint on the amount of domestic money holdings is imposed.[1] In this framework, money demand indeterminacy occurs only when money supplies evolve at the same rate. We show, however, that if money growth rates evolve in such a way that currency substitution becomes optimal, there will be a period of time during which the domestic inflation adjusts to equal the foreign inflation rate. We also show that the demand for foreign currency does not necessarily decline when a disinflation program is implemented in a dollarized economy. If, however, reversibility occurs (partial or total), the demand for foreign currency will decline smoothly, contrasting with the once-and-for-all change in the inflation rate.

The remainder of the paper is organized as follows. In the next section, we present the model. In Section 3, we discuss the dynamics of inflation and currency substitution. In Section 4, we discuss the possible outcomes of a disinflation program. Conclusions are presented in Section 5.

2. The model

Consider a small open economy that operates under flexible exchange rates and perfect capital mobility. Time is continuous. There exists only one international indexed bond and one consumption good, and purchasing power parity holds instantaneously. The economy is inhabited by a fixed number N of atomistic individuals who are blessed with perfect foresight and infinite life. Individuals maximize their lifetime utility function subject to a wealth constraint. For simplicity, in each instant individuals receive a fixed real income, and the real return of the indexed bond is equal to the rate of time preference. Transactions may be carried out by using both domestic and foreign money, the two currencies being intrinsically useless and perfect substitutes as means of payment. Foreign residents do not hold domestic money balances.

The consumption good may be purchased either with P units of domestic currency (M) or P^* units of foreign currency (F). The liquidity constraint requires purchases to be made with money held at the time of the transaction and broad money velocity is set equal to one. In real *per capita* terms:

$$m_t + f_t = 1 \tag{1}$$

where $m_t = M_t/NP_t$ and $f_t = F_t/NP_t^*$ are the home country per capita holdings of domestic and foreign real money balances at time t, and 1 refers to the normalized per capita consumption level.[2]

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[1] The rationale is that there are transactions, such as tax payments and transactions involving the government, that in general cannot be settled in foreign currency. Empirically, stubbornness of domestic money has been observed, even during extremely high inflation rates, such as the German hyperinflation (Giovannini and Turtelboom, 1994).

[2] Inflation taxes impact on the consumption level as a wealth effect. Since the real interest rate is equal to the rate of time preference, the optimal consumption is constant over time, so that the normalization above does not involve loss of generality.

To capture the existence of institutional limits to dollarization (e.g. transactions involving the government or closely monitored by it, that cannot be settled in foreign currency), it is assumed that the amount of transactions carried out with domestic currency cannot fall below a minimum level, $\bar{m}$. This assumption, together with (1) and the observation that money is dominated by an interest-bearing bond, implies that, for all t:

$$\bar{m} \leq m_t \leq 1. \tag{2}$$

When agents decide upon their cash holdings, they take the inflation path as given, because individually they cannot change the price level. The money demand path that achieves the highest consumption level is the one that minimizes the wealth erosion caused by the inflation taxes. Since domestic and foreign currencies are perfect substitutes and (2) defines a closed control set, the per capita demand for domestic money balances becomes:

$$m_t = \begin{cases} 1 & if \quad \pi_t < \pi^* \\ any & if \quad \pi_t = \pi^* \\ \bar{m} & if \quad \pi_t > \pi^* \end{cases} \tag{3}$$

where $\pi_t = \dot{P}_t/P_t$ and $\pi_t^* = \dot{P}_t^*/P_t^*$. This is a common arbitrage condition, stating that the domestic currency holdings of a representative individual will be respectively 1, any allowed amount or $\bar{m}$, as the domestic inflation is lower, equal to or higher than the foreign inflation rate. An implication is that, for any currency substitution to occur in this model, the domestic and foreign inflation rates must be equal, because only in that case will individuals hold a positive amount of both currencies.

The nominal money supply is assumed to be a continuous function of time. Denoting its rate of change by $\mu_t = \dot{M}_t/M_t$, the real money stock evolves according to:

$$\dot{m}_t = (\mu_t - \pi_t)m_t. \tag{4}$$

The paths of domestic inflation and real money balances are determined such that the aggregate real money demand and supply are equal. Due to the small country assumption, the domestic demand for foreign currency does not impact on the foreign inflation rate.

Equilibrium in this model may be defined in the following way:

Definition 1. *(monetary equilibrium). An equilibrium is given by a time-continuous function m_t for the individual agent and a function π_t for the whole economy such that, given the nominal rate of money growth, μ_t, eqs. (3) and (4) are continuously satisfied.*

In Definition 1, the real value of domestic money balances is required to be continuous over time because the nominal money supply is time-continuous, and jumps in the price level are ruled out by the assumptions of perfect foresight and no arbitrage.

In the following discussion, it will be useful to take as a benchmark the trivial solutions corresponding to time-invariant paths of μ:

Definition 2. *(trivial equilibria). When $\mu_t = \mu$ for all t, the equilibrium path of m_t is time-invariant and given by: (i) $m_t = \bar{m}$ if $\mu > \pi^*$; (ii) $m_t = 1$ if $\mu < \pi^*$; (iii) m_t any in (2) if $\mu = \pi^*$.*

To see this, we first note that any path with $\dot{m} \neq 0$ cannot be an equilibrium. Indeed, given the money demand (3), any non-horizontal path of m would only be possible if $\pi = \pi^*$. However, from eq. (4), the only way to achieve this when $\mu = \pi^*$ (case iii) is with a constant money demand. By the same reasoning, the equality of inflation rates in case (i) [(ii)], would require the money demand to grow [decline] exponentially forever, which is ruled out by (2). Hence, only horizontal money demand paths are allowed and, in these cases, we know from (4) that the domestic inflation rate is equal to the rate of money creation.

Since in cases (i) and (ii) domestic and foreign inflation rates differ, the optimal money demands are corner solutions. In case (iii), however, inflation rates are equal, so that any level of m is optimal from the individual point of view. In this case, the two currencies are perfect substitutes on both the demand and the supply side and, as in Kareken and Wallace (1981), there is a monetary equilibrium for each possible (time-invariant) value of the exchange rate, P/P^*.

3. The dynamics of inflation and currency substitution

In this section, we explore the adjustment dynamics of the model when the money supply path is such that individuals find it optimal to alter their cash-holdings.

Proposition 1. *(dynamics of inflation and currency substitution). If the domestic rate of money growth increases linearly over time, according to $\mu_t = \mu + \sigma t$, with $\sigma > 0$ and $0 < \mu < \pi^*$, then the equilibrium paths of m and π are:*

$$m_t = \begin{cases} 1 & t < \tilde{t} \\ \bar{m}\exp\left[\dfrac{\sigma}{2}t^2 - (\pi^* - \mu)t + \dfrac{(\pi^* - \mu)^2}{2\sigma}\right] & \tilde{t} \leq t < \bar{t} \\ \bar{m} & t \geq \bar{t} \end{cases} \tag{5}$$

and

$$\pi_t = \begin{cases} \mu_t < \pi^* & t < \tilde{t} \\ \pi^* & \tilde{t} \leq t \leq \bar{t} \\ \mu_t > \pi^* & t > \bar{t} \end{cases} \tag{6}$$

where $\bar{t} = \frac{\pi^* - \mu}{\sigma}$ and $\tilde{t} = \bar{t} - \sqrt{\frac{-2\ln\bar{m}}{\sigma}}$.

Proof: The third line of eq. (5) is equivalent to the trivial solution (i) in Definition 2. The intermediate segment is obtained using the non-arbitrage condition $\pi_t = \pi^*$ in (4) and solving for m_t, imposing $\lim_{t \to \bar{t}^-} m_t = \bar{m}$. $\tilde{t}$ is such that $m(\tilde{t}) = 1$.

To interpret Proposition 1 we refer to Fig. 1. We depict in the lower diagram the inflation rate (heavy line) and the rate of money growth (thin line), and in the upper diagram the money demand. Given the inflation path, individuals decide to use only domestic currency in the first segment, and the maximum allowed level of foreign currency in the third segment. In the intermediate segment, currency substitution takes place and the adjustment path is such that domestic and foreign inflation rates are equal. If the money demand declined slower, or faster, the inflation differential would not be zero, and no continuous path could be found such that the individual money demands and supply were continuously satisfied.
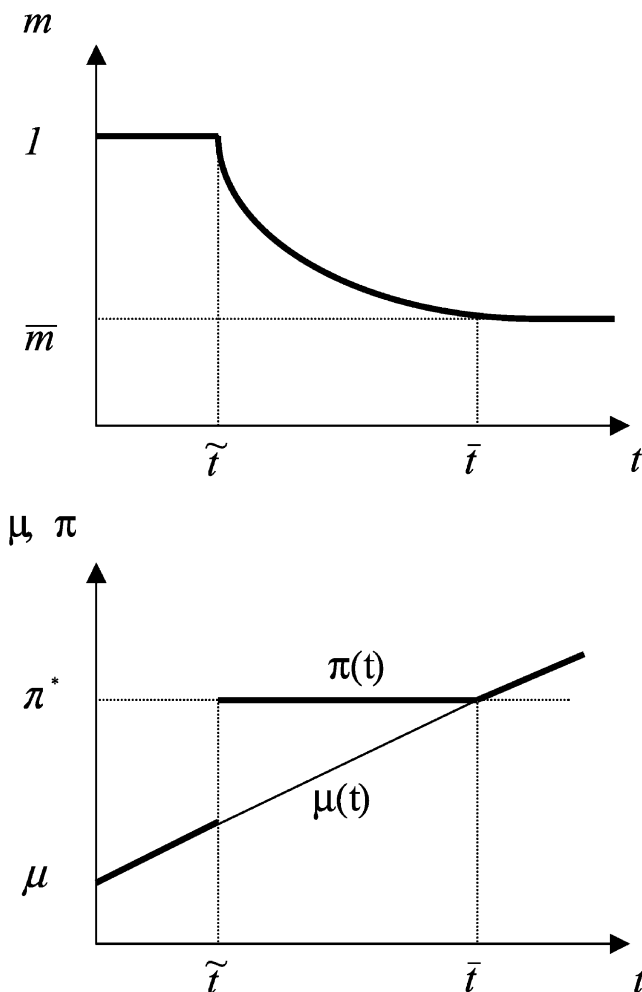

Fig. 1. The dynamics of inflation and currency substitution.

Due to the upper limit in (2), individuals have to wait for the right moment, $\tilde{t}$, to start the currency substitution process. The adjustment period, $(\bar{t} - \tilde{t})$, is shorter the higher is the minimum allowed proportion of domestic balances, $\bar{m}$, and the faster is the increase in the rate of money growth, σ.

4. The dynamics of a disinflation program

To investigate the dynamics of a disinflation program, assume that initially the domestic inflation rate is higher than the foreign inflation rate and that the initial level of dollarization is $f = 1 - \bar{m}$. The monetary authorities are assumed to reduce linearly the rate of money growth until it reaches the foreign inflation rate. When this is so, the following proposition holds:

Proposition 2. *(dynamics of a disinflation program). If the domestic rate of money growth follows* $\mu_t = \begin{cases} \mu - \sigma t & t < \bar{t} \\ \pi^* & t \geq \bar{t} \end{cases}$, *where* $\mu > \pi^*$ *and* $\bar{t} = \frac{\mu - \pi^*}{\sigma}$, *then there exist infinity of equilibrium paths for m and π given by:*

$$
m_t = \begin{cases} \bar{m} & t < \tilde{t} \\ m(\bar{t})\exp\left[-\dfrac{\sigma}{2} t^2 + (\mu - \pi^*)t - \dfrac{(\mu - \pi^*)^2}{2\sigma}\right] & \tilde{t} \leq t < \bar{t} \\ m(\bar{t}) & t \geq \bar{t} \end{cases} \tag{7}
$$

and

$$
\pi_t = \begin{cases} \mu - \sigma t > \pi^* & t < \tilde{t} \\ \pi^* & t \geq \tilde{t} \end{cases} \tag{8}
$$

where $m(\bar{t})$ is any in (2) and $\tilde{t} = \bar{t} - \sqrt{\dfrac{-2\ln(\bar{m}/m(\bar{t}))}{\sigma}}$.

Proof: Identical to Proposition 1, except that in this case there is an equilibrium for every possible (unchanging) value of m after $\bar{t}$.

This case has multiple equilibria because after the disinflation program the two currencies become perfect substitutes on the supply side, as in the trivial solution (iii) of Definition 2.[3] Hence, there exists a continuum of adjustment paths, corresponding to each possible level of dollarization in the new steady state.

To illustrate Proposition 2 we refer to Fig. 2. The figure has two panels, (a) and (b), each similar to Fig. 1. In panel (a) we depict the extreme case in which there is no decline in the dollarization level. In this case, the domestic inflation rate is always equal to the rate of money growth. In panel (b) we depict two possible

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[3] Of course, the indeterminacy occurs only when the disinflation program reduces the rate of money growth exactly to the level of foreign inflation. However, the indeterminacy can easily be extended to a range of money supply paths, introducing a simple network externality in the use of money. Such an extension is available from the author upon request.
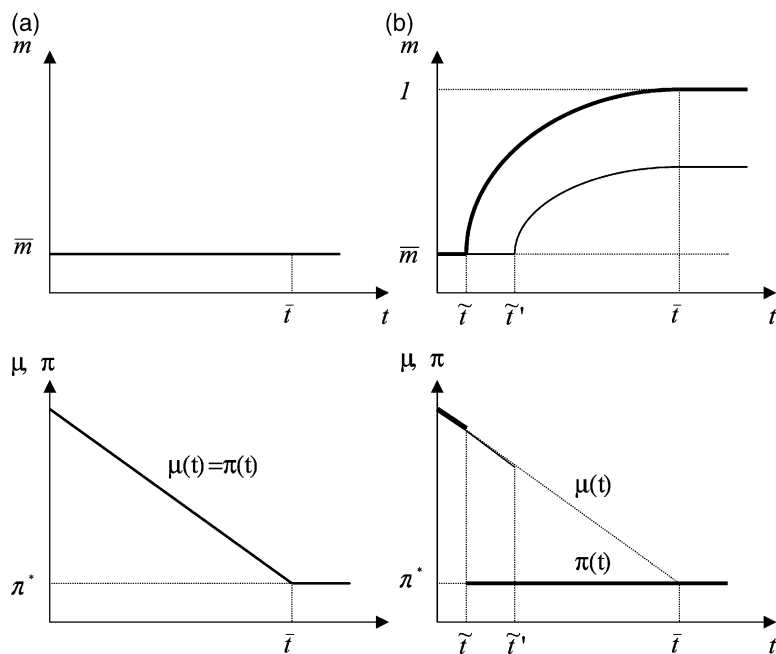
Fig. 2. The dynamics of a disinflation program.

paths where the disinflation program leads to a decline in the level of dollarization. The thin line depicts a case in which some dollarization remains at the end of the adjustment program. The heavy line corresponds to the extreme case in which the foreign currency is completely abandoned, that is, $m(\bar{t}) = 1$. In both cases the demand for domestic money rises along the adjustment period, implying an inflation rate lower than the rate of money creation and equal to the foreign inflation rate, as required by the non-arbitrage condition. Since in the thin line case the cash adjustment is smaller, the currency substitution process starts later ($\tilde{t}' > \tilde{t}$). Thus, unlike the situation in Proposition 1, the length of the adjustment period ($\bar{t} - \tilde{t}$) is not controllable by the monetary authorities. Because of the indeterminacy, the policy parameter, σ defines only an upper bound on the length of the adjustment period.

If possible, a planner would choose the equilibrium path leading to $m = 1$ (heavy line in panel b), because in that case all seigniorage revenues would accrue to the domestic economy. However, in a decentralized economy, there is no way of assuring that this equilibrium will occur[4]

In all equilibria described by Proposition 2, the demand for foreign currency fails to respond linearly to changes in the inflation rate, and this accords with the

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[4] Note that any small barrier to change, such as a lump sum switching cost, would lead to the less favorable outcome, depicted in panel (a).

empirical evidence. The equilibrium depicted in panel (a) captures the extreme case in which the demand for foreign currency does not decline at all. The remaining equilibria account for partial and full reversibility. In all of these cases, the demand for foreign currency declines smoothly, following the once-and-for-all adjustment of the inflation rate. The fall in the inflation rate does not help to predict, however, the length of the adjustment period or the extent of de-dollarization that will be achieved in the long run. Since the maximum length of the adjustment period depends on the policy parameter σ, a wide range of de-dollarization patterns may be observed after the fall in the inflation rate.

5. Conclusions

In this paper we investigate the dynamics of money demand and inflation in a small open economy, where domestic and foreign currencies are perfect substitutes as means of payment. The main property of this model is that, if the path of money supply is such that individuals find it optimal to change the currency in which transactions are settled, there will be an adjustment period during which the domestic inflation rate adjusts to equal the foreign inflation rate. This implies that, during the currency substitution process, the behavior of money demand is not accounted for by changes in the current inflation rate. In the context of this model, a temporary increase in the rate of money creation gives rise to a demand for foreign currency that does not necessarily decline when the inflation rate comes down. If, however, reversibility occurs (partial or total), the demand for foreign currency will decline smoothly, contrasting with the once-and-for-all change in the inflation rate. Therefore, the model is able to explain the failure of econometric studies in identifying stable money-demand relationships in dollarized economies.

The model captures the different patterns of dollarization identified in the literature, namely reversibility in some cases and non-reversibility in others. This does not depend on any ad-hoc dollarization cost function. It is, instead, an obvious consequence of the assumption of perfect substitutability. Reversibility in this context is assured if the domestic currency becomes a better alternative than the foreign currency. Such an event, however, may not be easy to achieve in most developing economies. In these cases, co-existence will be the more likely outcome of a successful stabilization plan, unless administrative actions are taken to banish the foreign currency.

It may be argued that the knife-edge equilibria discussed in this paper are not realistic. It is true that in the real world agents do not switch between currencies in response to small inflation differentials. As mentioned above, however, the model can easily be extended so as to better resemble reality, introducing some form of dollarization costs. But there is nothing particularly new in such an extension that makes it more attractive than the version discussed in this paper. On the contrary, by focusing on the simpler model, we stress the main point that there are more fundamental forces driving the behaviour of money demand than dollarization costs, which are not even necessary to explain the stylized facts.

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