Macroeconomic Dynamics, 2016, Page 1 of 30. Printed in the United States of America.
doi:10.1017/S1365100515000425

MODEL-BASED VS. PROFESSIONAL FORECASTS: IMPLICATIONS FOR MODELS WITH NOMINAL RIGIDITIES

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We compare model forecast error statistics with forecast error statistics of professional forecasts. We look at a standard sticky-prices–wages model, concluding that it delivers too strong a theoretical forecastability of the variables under scrutiny, at odds with the data (professional forecasts). We argue that the lack of compatibility between the model and professional forecasts results from trying to fit inflation (which is probably nonstationary) to a model that assumes inflation is stationary. A modified version of the model, one with a varying inflation target, delivers a better fit in terms of forecastability.

Keywords: Survey of Professional Forecasters, DSGE Models, Nominal Rigidities

1. INTRODUCTION

Despite tremendous efforts over the past few decades, macroeconomic forecasting is still a difficult task. Simple models prove hard to beat, sophisticated statistical methods provide marginal (if any) improvements at long horizons, and for most variables forecast accuracy is low. The same difficulty characterizes professional forecasts (say, from the Philadelphia Survey of Professional Forecasters, henceforth SPF, or the Federal Reserve Green Book, henceforth Green-Book). We argue that this degree of forecastability ought to be considered a feature of realistic model economies.

The authors are grateful to Valerio Ercolani, António Antunes, Nikolay Iskrev, Pedro Teles, Isabel Correia, Bernardino Adão, Andrew C. Harvey, Petra Geraats, Mikhail Pranovich, Eva Ortega, Gabriel Perez-Quiros, James Costain, Samuel Hurtado, Juan Francisco Jimeno, Javier Ortega, and Michael Ben-Gad for insightful comments and suggestions, as well as to seminar participants at the University of Cambridge, the Banco de Portugal, the Banco de España, the University of Birmingham, the University of Liverpool, and City University London. The usual disclaimer applies. Most research was done while João Tovar Jalles was a Visiting Researcher at the Banco de Portugal, whose hospitality is greatly appreciated. The views expressed are those of the authors and do not necessarily represent those of the Banco de Portugal, the OECD, or its member states. Address correspondence to: João Valle e Azevedo, Banco de Portugal, Research Department, Av. Almirante Reis, 71-6th floor, 1150-012 Lisboa, Portugal; e-mail: azevedojv@gmail.com

 1365-1005/16

We view professional forecasts as the best publicly available proxy for the forecasts produced by well-informed agents in the economy, providing a natural benchmark against which to confront the forecast error statistics obtained in a model economy. We focus on SPF forecasts to measure forecastability because there is clear evidence that they rank very well compared with the Green-Book, with more recent survey-based forecasts (such as the Blue Chip and Consensus Forecasting), and with various statistical methods [see, e.g., Ang et al. (2007), Rubaszek and Skrzypczynski (2008), Baghestani (2009, 2012), Kolasa et al. (2012), Stark (2010), Faust and Wright (2012), or Valle e Azevedo and Pereira (2013)].[1]

Departing from the common practice of comparing second moments implied by the model with second moments observed in the data, we compare model forecast error statistics with the same statistics obtained with professional forecasts. If our model economy were an AR(1) with known autoregressive parameter (ρ) and known variance of the error term (σ^2), we would compute the h-step ahead forecast error variance (assuming a mean square criterion to pick the point forecast) as $\sigma^2 \frac{1-\rho^{2h}}{1-\rho^2}$, which would then be compared with the variance of the forecast error obtained with professional forecasts. Small differences in forecast accuracy in the two worlds (model and professional) would be interpreted as a sign that the model was able to replicate an important dimension of actual data. This is in the spirit of Rotemberg and Woodford (1996), who compare the forecastability obtained with a VAR with that found in a real business cycle (RBC) model and argue on the potential of the exercise to reveal sources of misspecification. The difference is that, instead of using statistical methods, we focus on the survey-based SPF forecasts. This amounts to taking up the challenge of Diebold and Kilian (2001, p. 668): "An interesting extension of this paper will be to use these [survey-based] forecasts to compute survey-based estimates of predictability, and to compare the survey-based and model-based estimates." We would add that the alternative of using statistical methods to compute predictability would entail a choice among endless options. Surely, one could always use standard univariate statistical methods. But, as Diebold and Kilian (2001, p. 668) put it, "The survey-based approach is of interest because the information sets used by actual forecasters are likely much richer than simple univariate histories. They are surely multivariate, for example, and they also contain hard-to-quantify subjective information. The survey-based approach does rely on a crucial and disputable assumption (optimality of reported forecasts), but so too does the model-based approach (adequacy of the fitted model)."

Among the many issues potentially raised by these comparisons, we concentrate on two main questions: Can the behavior of professional forecasts be reconciled with the forecastability implied by a standard dynamic stochastic general equilibrium (DSGE) model with nominal rigidities (the Smets and Wouters 2007 model)? Because we find too much forecastability in the model relative to the data (professional forecasts), the answer to this question seems to be "hardly." We argue that one critical feature of the model, namely the existence of a fixed steady-state inflation rate, is the main source of this disagreement. In this setup the estimated model fits (highly persistent) inflation by attributing to nominal rigidities a disproportionate role, which induces forecastability in all the model variables. This leads naturally to a second question: would a modified version of the model, e.g., one with a varying inflation target, deliver a better fit in terms of forecastability? The broad answer seems to be "yes."

The motivation for these questions follows from the observation that for the growth of real variables such as consumption, investment, or output, there is little predictability in the data (professional forecasters). Such predictability is below that obtained (theoretically) in the standard Smets and Wouters (2007) world, but not out of bounds. But, additionally, and strikingly, in the case of GDP inflation and focusing on the sample 1984q1–2009q2, professional forecasters can only account for 16% of its standard deviation at one quarter horizon (4% at two quarters), whereas the standard Smets and Wouters model (under a standard "Great Moderation" parameterization) would account for 51% at one quarter horizon (31% at two quarters). In contrast, a version of the model with a varying inflation target delivers variances of forecast errors that are closer to those obtained by professional forecasters.

The remainder of the paper is organized as follows: Section 2 analyzes, for a host of variables, the predictive power of professional forecasts. Section 3 confronts these facts with what obtains in the DSGE model (focusing on fewer variables). Section 4 concludes.

2. PROFESSIONAL FORECASTS: HOW MUCH THEY DELIVER

2.1. Data

We analyze 14 macroeconomic indicators from the SPF, namely, nominal GNP/GDP, real GNP/GDP, industrial production index—total, real personal consumption expenditures—total, GDP deflator, Consumer Price Index, real gross private domestic investment—residential, real gross private domestic investment—nonresidential, real government consumption and gross investment—state and local, real government consumption and gross investment—federal, housing starts, unemployment, 3-month T-bills, 10-year T-bonds.[2] We look only at point forecasts and define these as the median forecasts in every release of the survey (results with the mean forecast are very similar and will not be reported).[3]

Our sample for SPF forecasts spans 1984q1–2009q2, except for T-bond data, which start in 1992q1. We thus focus on the Great Moderation and avoid to a certain extent quibbles related to the likely changes in the processes generating the data (and their forecastability) in the postwar period.[4] All data are first aggregated quarterly when necessary (to be consistent with the variables forecast in the SPF), and except for unemployment and interest rates, all data are in growth rates. Please refer to Appendix A for all the details.

2.2. Methodology

We assess the predictive power of SPF forecasts by measuring simply their performance relative to an estimate of the unconditional means of the variables analyzed. More specifically, we compute the average of each variable from 1982q4 through to 1984q1$-h$ quarters, for $h = 1, \ldots, 5$.[5] This is our benchmark forecast for 1984q1, denoted "real-time average." We then compute the average from 1982q4 through to 1984q2$-h$, $h = 1, \ldots, 5$, to forecast 1984q2, and so forth until 2009q2, i.e., with an expanding window of observations. Given the possibility of a unit root in interest rates and inflation, it is sensible to consider also a random walk forecast in these cases.

It should be noted that to properly compare the benchmark forecasts with SPF forecasts, we re-label the forecast horizon of SPF forecasts so that the implicit information sets with each method approximately coincide. This means that the -step-ahead SPF forecasts, with $0 \leq h \leq 4$, will be denoted as $(h+1)$-step-ahead forecasts. For more details on the timing of the forecasts see Appendix A.

We then compare forecast accuracy by computing the ratio of the root-mean-square forecast error (RMSFE) of SPF forecasts to the RMSFE of the benchmark forecast (real-time average or random walk forecast). Following Fair and Shiller (1989), we also run the following forecast encompassing regression:

$$y_{t+h} = \alpha + \beta_0 f_{t+h}^{\text{naive}} + \beta_1 f_{t+h}^{\text{SPF}} + \varepsilon_{t+h}, \qquad (1)$$

where y_{t+h} is the observation of the forecast variable, f^{real} is the naive (real-time average or random walk) forecast, f^{SPF} is the SPF forecast, and ε_{t+h} is a (most likely serially correlated) regression error. Obviously, if $\beta_1 \neq 0$, then SPF forecasts add information relative to the naive forecast and to the constant term.

2.3. Results—Forecast Accuracy

Table 1 contains the ratio of the RMSFE obtained with professional forecasts to that obtained with the benchmark forecast (real-time average or random walk), as well as the estimate of β_1 resulting from OLS estimation of equation (1) at different forecast horizons. The main conclusions follow:

- By looking at the significance of β_1, we conclude that forecasts of CPI and GDP inflation, unemployment, Federal spending, and T-bill rate add signal relative to the real-time average up to $h = 5$. For forecasts of private residential and nonresidential fixed investment as well as for T-bonds, β_1 is significant up to $h = 4$. However, for the remaining variables under scrutiny, SPF forecasts add signal relative to the real-time average only up to $h = 3$ ($h = 2$ in the case of housing starts).
- The relative (to the real-time average) RMSFE for SPF forecasts is clearly less than one for all horizons only in the case of unemployment, interest rates, and residential investment. For all the other variables this ratio is only small (say, less than 0.75) for $h = 1$ in some instances and mostly close to one for $h \geq 3$. Still, in the case of nominal GDP, nonresidential private investment, and housing starts, we do find ratios

TABLE 1. Forecast performance of the SPF

	$h = 1$		$h = 2$		$h = 3$		$h = 4$		h=5	
	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$
Horizon	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
RMSFE relative to										
Nominal GNP/GDP										
Real-time avg.	0.62	1.13***	0.75	1.15***	0.82	0.66**	0.84	0.11	0.84	−0.32
Real GNP/GDP										
Real-time avg.	0.73	1.10***	0.83	1.08***	0.90	0.62*	0.91	0.17	0.91	−0.14
Industrial production index—total										
Real-time avg.	0.66	1.25***	0.86	1.24***	0.93	0.85*	0.94	0.26	0.89	0.26
Real personal consumption expenditures—total										
Real-time avg.	0.75	1.05***	0.83	1.39***	0.90	1.18***	0.93	0.25	0.91	0.55
Consumer price index										
Real-time avg.	0.82	0.83***	0.80	1.36***	0.91	0.84***	0.92	0.76***	0.94	0.66***
Random walk	0.71	0.77***	0.67	1.34***	0.83	0.70***	0.72	0.85***	0.78	0.65***
Real private fixed investment—residential										
Real-time avg.	0.54	1.57***	0.64	1.74***	0.68	1.45***	0.69	0.73**	0.69	0.51
Real private fixed investment—nonresidential										
Real-time avg.	0.66	1.31***	0.77	1.61***	0.83	1.63***	0.84	1.44***	0.86	0.44
Real government consumption and gross investment—state and local										
Real-time avg.	0.93	1.39***	0.92	1.98***	0.92	1.79***	0.97	0.66	0.96	0.48

TABLE 1. Continued

	$h = 1$		$h = 2$		$h = 3$		$h = 4$		h=5	
	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$	RMSFE SPF	$\widehat{\beta}_1$
Horizon	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					Real government consumption and gross investment—federal					
Real-time avg.	0.91	0.68***	0.89	1.03***	0.95	0.85***	0.87	1.34***	0.89	1.15***
					Housing starts					
Real-time avg.	0.65	1.16***	0.81	0.99***	0.88	-0.003	0.83	0.14	0.79	−0.01
					GDP deflator					
Real-time avg.	0.81	0.98***	0.91	1.00***	1.00	1.01***	1.11	0.92***	1.11	0.94***
Random walk	0.91	0.51***	1.00	0.36**	1.07	0.31*	1.21	0.19	1.09	0.25*
					Unemployment rate					
Real-time avg.	0.10	1.05***	0.22	1.15***	0.32	1.24***	0.43	1.23***	0.50	1.16***
					10-year Treasury constant maturity rate (post-1992Q1)					
Real-time avg.	0.14	0.96***	0.44	0.77***	0.60	0.61***	0.70	0.40***	0.76	0.19
Random walk	0.43	1.11***	0.89	0.92***	0.99	0.71***	1.02	0.67**	1.04	0.61
					3-month Treasury bill					
Real-time avg.	0.06	0.98***	0.22	0.96***	0.37	0.91***	0.51	0.83***	0.62	0.66***
Random walk	0.31	1.16***	0.63	1.27***	0.77	1.36***	0.86	1.49***	0.90	1.46***

Notes: The evaluation sample is 1984q1–2009q2, except for T-bond data, for which the evaluation period starts in 1992q1. Odd columns report the ratio of the RMSFE obtained with SPF predictions to the one obtained with the real-time average or the random walk forecast. Even columns report the coefficient resulting from OLS estimation of Equation (1) for the comparison of the real-time average (or random walk) and SPF forecasts.

*,**,*** Significance at 10%, 5%, and 1% levels, respectively (or rejection of the null hypothesis that β_1 is zero; Newey–West robust standard errors with lag window equal to 4 are employed).

around 0.85 for $h \geq 3$. It is also worth noticing the relative RMSFEs greater or equal to 1 for forecasts of GDP inflation when $h \geq 3$.

In short, this simple exercise shows that for most variables a real-time average seems a hard-to-beat forecast (even) at short horizons. Regarding residential investment, unemployment, nominal interest rates, and inflation, professional forecasts do contain relevant information beyond this crude benchmark forecast, although in the case of inflation the average forecast accuracy (or RMSFE) differs little from that of the benchmark. Regarding the random walk forecast (computed for the two inflation measures and interest rates), we note that, except for CPI inflation, the performance is superior to that of the real-time average but still (most often) inferior to that of SPF forecasts. Our crucial observation remains valid: SPF forecasts lose their value added quite fast vis-à-vis these rough benchmarks.

2.4. Discussion

Quantitative macroeconomic models are often judged according to their capacity to fit some dimensions of the data. For example, they are commonly required to deliver steady-state ratios, volatilities, and correlations that are close to what one observes in the data. In this paper we take the view that the forecast error statistics obtained by well-informed agents in the actual economy (professional forecasters) should at the very least resemble those generated *theoretically* by a realistic DSGE model. For example, if the model delivers a RMSFE (relative to the standard deviation) for output growth at 1 quarter horizon equal to 0.3, whereas professional forecasters (data) attain 0.8, we view this as an indication that the model delivers a strong forecastability that is at odds with the data.

In our view, the preceding comparisons have the potential, along with other model validation devices, to inform theory. Obviously, everyone would agree that many features of any model are at odds with the data. We will highlight one such clear mismatch, arguably driven by the implications of nominal rigidities on the forecastability of real variables and of inflation.

3. HOW DOES A STANDARD DSGE MODEL FORECAST?

We move now toward the core of the paper, comparing the SPF with the theoretical forecast performance of the medium-scale model analyzed and estimated in Smets and Wouters (2007; henceforth SW07), based on Smets and Wouters (2003) and Christiano et al. (2005).

3.1. The Model

The model has many of the now popular features in the literature, including monopolistic competition in the goods and labor markets, ingredients aimed at improving the fit of the model to observables such as habit formation in consumption, investment adjustment costs, variable capacity utilization, and, crucially, nominal frictions in the form of Calvo sticky prices and wages, along with partial backward-looking indexation. Monetary policy follows a Taylor rule. Seven shocks are included (total factor productivity, investment productivity, monetary policy, government spending, and risk premium following an AR(1) process, along with price and wage markup shocks following an ARMA(1, 1) process), as well as seven observables, which we include in the vector $\mathbf{y}_t$: output, investment, consumption, wages (all in log differences), and inflation, nominal interest rate, and (log of) hours. The model equations can be found in Appendix B.

We start by solving a first-order log-linear approximation of the model using the algorithm of Swanson et al. (2005). We then build the state-space representation of the solution, including measurement equations linking variables in the model to observed variables:

$$\mathbf{s}_t = T(\theta)\mathbf{s}_{t-1} + R(\theta)\varepsilon_t \text{ (Policy function)},$$

$$\mathbf{y}_t = Z\mathbf{s}_t \text{ (Measurement equation)},$$

where $\mathbf{s}_t$ is the state vector including all the endogenous variables with initial distribution assumed to be $\mathcal{N}(\mathbf{s}_0, P_0)$, the stationary distribution, and ε_t is the vector of exogenous shocks, assumed normally distributed with mean zero and variance $Q(\theta)$. The matrices $T(\theta)$ and $R(\theta)$ also depend on the vector θ of deep parameters of the model, and the matrix Z selects the elements from $\mathbf{s}_t$.[6] We use exactly the same data treatment as in SW07, implying that the match between the model's variables and the SPF's counterparts is not perfect. Specifically, SW07's observables for real output, consumption, investment (nonresidential), and wages are expressed in per capita terms (working age population) and the nominal interest rate is measured with the Federal funds rate (quite close to the 3-month T-bill rate from SPF, nonetheless). The inflation measure in the model is GDP inflation (i.e., perfect match with SPF), whereas (minus) SPF's unemployment, although following hours closely, surely drifts somehow from the concept in the model. Finally, we use SW07's calibrated and estimated parameters (mode of the posterior distribution), which were obtained using data from 1984q1 through 2004q2; see details in Appendix B. We choose this sample to avoid quibbles regarding the onset of the "Great Moderation" and likely changes in monetary policy within the period starting in 1966q1 (SW07's beginning of the sample).

We analyze the theoretical forecast performance of various versions of SW07: the original one featuring price and wage rigidities, a version with only wage rigidities (with and without indexation of wages to price inflation), a version with no indexation of prices, and another where we shut down all the rigidities, i.e., the flexible prices or RBC version (where inflation and the nominal interest rate are not included). All the other structural parameters are kept fixed. To compute the theoretical forecastability of the model, we take as forecasts of the observables at $T + h$, conditional on information at time T, the conditional expectations $\widehat{\mathbf{y}}_{T+h|T} := E[\mathbf{y}_{T+h}|\mathbf{y}_{T,}\mathbf{y}_{T-1}, \ldots, \mathbf{y}_1]$, where T is the forecast moment and h is the forecast horizon. The theoretical covariance matrix of the forecast errors, or

TABLE 2. Theoretical forecast models (versions of SW07)

Model designation	Price rigidities	Price indexation	Wage rigidities	Wage indexation	Observables
RBC	No	No	No	No	C, I, Y, W, H
NK—only W rigid.	No	No	Yes	No	C, I, Y, W, R, Π, H
NK—only W rigid. + W index.	No	No	Yes	Yes	C, I, Y, W, R, Π, H
NK—no P index.	Yes	No	Yes	Yes	C, I, Y, W, R, Π, H
NK	Yes	Yes	Yes	Yes	C, I, Y, W, R, Π, H

Notes: *C*, consumption; *I*, investment; *Y*, output; *W*, wages (all in log differences); *R*, nominal int. rate; *H*, hours; Π, inflation (in levels).

$\Omega_{T+h|T} := E[(\mathbf{y}_{T+h} - \widehat{\mathbf{y}}_{T+h|T})(\mathbf{y}_{T+h} - \widehat{\mathbf{y}}_{T+h|T})']$, can be obtained with the standard Kalman recursions. Finally, we set $T = 160$ (thinking of 40 years of quarterly data). For all practical purposes, $\Omega_{T+h|T} \approx \Omega_h$ for $T \geqslant 160$.

3.2. Theoretical Forecastability: The Smets and Wouters (2007) Model

Now, recall that results in Section 2 suggest that for most real variables (in particular, output, nonresidential investment, and consumption), professional forecasts add little relative to an estimate of the unconditional mean (real-time average). In contrast, professional forecasts of unemployment and the nominal interest rate are still clearly superior to the real-time average after one year, whereas for inflation (CPI and GDP deflator), we found that SPF forecasts are not encompassed by the real-time average, but at longer horizons their average forecast accuracy differs little. The question we address here is whether this behavior is shared theoretically by the model economy.

To do so, we derive the model RMSFEs at various horizons (just the square root of the diagonal elements of the matrix $\Omega_{T+h|T}$ defined before), normalized by the standard deviation of the variables. We compare this with the normalized (by the standard deviation of the observed variables) RMSFEs obtained by the SPF over the period 1984q1–2009q2.[7] Table 2 summarizes the various versions of the SW07 model considered, and Table 3 reports the normalized RMSFEs. Besides comparing the RMSFEs, we include a summary measure of fit. We report the geometric mean of the differences between the normalized RMSFEs obtained with the SPF and those obtained with the model (i.e., differences between the SPF entries in Table 3 and the model's entries). All the horizons (from $h = 1$ to $h = 5$) and variables considered are pooled together. We report this measure of fit considering all the model variables except wages (as wages are not forecast in the SPF) as well as for the group of variables C, I, Y, H.[8] The latter group allows us to compare the fit with that of the RBC version (where inflation and nominal interest rates are not pinned down).

TABLE 3. Theoretical root-mean-square forecast error of models vs. SPF

		Horizon								
Variable	Model	1	2	3	4	5	6	8	12	20
Real GDP growth	SPF	0.76	0.88	0.97	0.99	1.02	—	—	—	—
	RBC	0.87	0.96	**0.98**	**0.98**	**0.98**	0.98	0.99	0.99	0.99
	NK—only W rigid.	**0.78**	**0.87**	0.91	0.93	0.94	0.94	0.95	0.96	0.97
	NK—W rigid. + W index.	0.73	0.82	0.87	0.91	0.92	0.93	0.93	0.94	0.97
	NK—no P index.	0.73	0.82	0.87	0.91	0.92	0.93	0.93	0.94	0.97
	NK	0.73	0.81	0.87	0.90	0.92	0.93	0.93	0.94	0.97
Consumption growth	SPF	0.80	0.89	0.97	1.01	1.00	—	—	—	—
	RBC	**0.77**	**0.90**	**0.94**	**0.94**	**0.94**	0.95	0.95	0.97	0.98
	NK—only W rigid.	0.73	0.84	0.90	0.94	0.94	0.95	0.95	0.97	0.98
	NK—W rigid. + W index.	0.65	0.78	0.86	0.91	0.94	0.95	0.95	0.96	0.98
	NK—no P index.	0.64	0.77	0.85	0.90	0.93	0.94	0.94	0.95	0.97
	NK	0.63	0.74	0.82	0.87	0.90	0.91	0.92	0.93	0.95
Invest. nonresid. growth	SPF	0.67	0.80	0.89	0.93	0.98	—	—	—	—
	RBC	0.82	0.92	0.93	**0.94**	**0.94**	0.95	0.97	0.99	1.00
	NK—only W rigid.	0.74	0.84	**0.88**	0.90	0.91	0.91	0.93	0.95	0.97
	NK—W rigid. + W index.	**0.68**	**0.79**	0.84	0.87	0.88	0.91	0.91	0.92	0.96
	NK—no P index.	**0.68**	**0.79**	0.84	0.86	0.88	0.90	0.91	0.92	0.96
	NK	**0.68**	**0.79**	0.83	0.86	0.87	0.89	0.90	0.92	0.95

Table 3. Continued

Variable	Model	Horizon 1	2	3	4	5	6	8	12	20
Hours	SPF (unemployment)	0.13	0.29	0.45	0.62	0.74	—	—	—	—
(exc. SPF)	RBC	0.52	0.67	0.74	0.78	**0.81**	0.82	0.85	0.88	0.95
level	NK—only W rigid.	0.24	0.34	**0.42**	**0.50**	0.57	0.63	0.73	0.86	0.95
	NK—W rigid. + W index.	**0.19**	**0.28**	0.36	0.44	0.51	0.58	0.71	0.86	0.95
	NK—no P index.	**0.19**	**0.27**	0.35	0.43	0.50	0.57	0.69	0.84	0.93
	NK	**0.19**	**0.27**	0.34	0.41	0.48	0.55	0.67	0.82	0.91
Wages	SPF	—	—	—	—	—	—	—	—	—
growth	RBC	0.70	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	NK—only W rigid.	0.91	0.95	0.96	0.96	0.96	0.97	0.97	0.98	0.99
	NK—W rigid. + W index.	0.91	0.93	0.93	0.93	0.93	0.94	0.94	0.96	0.97
	NK—no P index.	0.93	0.95	0.95	0.95	0.96	0.96	0.97	0.99	1.00
	NK	0.91	0.93	0.94	0.94	0.95	0.95	0.97	0.98	0.99
Inflation	SPF	0.84	0.96	1.06	1.19	1.20	—	—	—	—
level	NK—only W rigid.	**0.73**	**0.87**	**0.92**	**0.95**	**0.97**	0.98	0.99	0.99	0.99
	NK—W rigid. + W index.	0.62	0.79	0.88	0.93	0.96	0.97	0.98	0.99	0.99
	NK—no P index.	0.60	0.77	0.86	0.91	0.94	0.95	0.97	0.97	0.97
	NK	0.49	0.69	0.81	0.88	0.92	0.94	0.96	0.97	0.97
	NK—univariate	0.50	0.71	0.82	0.89	0.92	0.94	0.96	0.97	0.97

TABLE 3. Continued

Variable	Model	Horizon 1	2	3	4	5	6	8	12	20
Int. rate	SPF	0.07	0.24	0.41	0.57	0.71	—	—	—	—
level	NK—only W rigid.	0.33	0.51	0.64	0.73	0.79	0.84	0.89	0.94	0.97
	NK—W rigid. + W index.	0.27	0.44	0.58	0.69	0.76	0.82	0.89	0.94	0.97
	NK—no P index.	0.26	0.43	0.57	0.67	0.76	0.81	0.89	0.94	0.97
	NK	**0.23**	**0.40**	**0.53**	**0.64**	**0.72**	0.78	0.87	0.92	0.97
Fit	RBC			—						
C, I, Y, R, Π, H	NK—only W rigid.			**0.075**						
	NK—W rigid. + W index.			0.087						
	NK—no P index.			0.092						
	NK			0.094						
Fit	RBC			0.058						
C, I, Y, H	NK—only W rigid.			**0.051**						
	NK—W rigid. + W index.			0.063						
	NK—no P index.			0.068						
	NK			0.075						

Notes: Forecastability of theoretical models vs. SPF. Normalized (or relative to the standard deviation of the variables) RMSFE at different forecast horizons. The standard deviation is that of the model variables in the case of model forecasts or that in the data in the case of SPF forecasts. SPF's evaluation period is 1984q1–2009q2. Bold entries represent, for each horizon and variable, the values closest to the SPF. *Fit* refers to the geometric mean of the differences between the normalized RMSFEs obtained with the SPF and those obtained with the model, with the horizons from $h = 1$ to $h = 5$ and the variables below Fit pooled together.

Several conclusions follow:

- Models with more nominal rigidities diplay stronger forecastability. In other words, the relative RMSFE decreases, almost always, as we move from the model with only wage rigidities (without indexation) to the model with price and wage rigidities (and indexation). The differences are not irrelevant across variations.
- Still focusing on models with nominal rigidities, we observe that for the nominal interest rate, inflation, and hours, there is strong predictability at short horizons. In the case of the New Keynesian (NK) model, the convergence of the rel. RMSFE toward 1 is rather slow, and after eight quarters it is still 0.67 for hours and 0.87 for the nominal interest rate. For consumption, output, and investment, convergence is faster. Wages are the least predictable variable, with a relative RMSFE starting around 0.91–0.93.
- The RBC version is silent with respect to inflation and the nominal interest rate, but for the remaining variables the convergence of the relative RMSFE is much faster. Still, in the case of hours, convergence of the RMSFE toward 1 is slow, even if at a level very clearly above that of the versions with nominal rigidities.

In short, and focusing on the original NK model, the analysis shows the existence of strong forecastability in the case of hours, inflation, and the nominal interest rate, even at very long horizons. For consumption, output, and especially investment, there is still some forecastability six quarters ahead, but not much beyond that horizon.[9]

Comparing these results with what obtains with SPF forecasts, we notice the stronger forecastability of models with nominal rigidities vis-à-vis the SPF, even if in several instances the differences are not out of bounds.[10] Another relevant takeaway is that, all in all, the best balance in the match between the SPF and model forecastability seems to be obtained by the model with fewer rigidities, or the NK—only W rigid. model (the same information is conveyed by the measure of fit we report). Importantly, the RBC fails to achieve this balance, mainly because of hours at short horizons.

Next, the differences between the model NK—only W rigid. and versions with more rigidities vis-à-vis professional forecasts are not irrelevant. Strikingly, in the case of GDP inflation, professional forecasters can only account for 16% of its standard deviation at $h = 1$ (4% at $h = 2$). Now, the model NK—only W rigid. would account for 27% of its standard deviation at $h = 1$ (13% at $h = 2$), whereas the standard Smets and Wouters model (NK version) would account for 51% at one quarter horizon (31% at two quarters)! This is a consequence of the degree of backward-looking behavior of inflation in the NK model, aimed at fitting the observed persistence of inflation. Moreover, once current inflation and its history are available, information about other variables is almost irrelevant to forming a close to efficient conditional expectation of inflation; see the NK-univariate specification in the case of inflation in Table 3. With only past inflation used to form the forecast, the model can still account for 50% of the standard deviation of inflation (29% at two quarters)! If the model is realistic, this implies that a forecaster would need only to nail the univariate representation of inflation in order to obtain a close to efficient forecast.

Finally, and importantly, the fact that the relative RMSFE for the SPF forecasts of inflation reaches 1.19 and 1.20 at $h = 4, 5$ may indicate that panelists are unable to estimate the unconditional mean of inflation, simply because it does not exist. Otherwise they could just pick that estimate at long horizons and guarantee a relative RMSFE close to 1.

3.3. Discussion

All in all, the preceding results suggest that the nominal rigidities apparatus of the original NK model produces strong theoretical forecastability of several variables. This seems at odds with the data. A simpler version with only wage rigidities (and no indexation) delivers a forecastability resembling more that of the SPF. In this simpler version, deviations from the steady state are less persistent, which translates into lower forecastability. On the other hand, the version we have analyzed with fully flexible prices and wages (RBC) results in too little theoretical forecastability. The most striking departure of this model from the data occurs with hours at short horizons, although for output and investment the differences are also non-negligible.

A natural question then arises: can these results reveal some sort of misspecification in the original model? In our view they can. What occurs with inflation seems revealing in this respect. In the original NK model we found strong forecastability of this variable, at odds with the data. Further, nailing the univariate representation of inflation would be enough to obtain a close to efficient forecast. But then, why have professional forecasters been unable to get close to this forecast? A possible answer is that there is not a fixed steady-state, or Fed target, inflation rate, even in a post-1984 sample. Add to this the fact that inflation is most often regarded as following an I(1) process, even in a post-1984 sample—see, e.g., Stock and Watson (2007)—not as a stationary process, as the model assumes. Now, under the assumption of a fixed steady-state inflation rate, the obvious way an estimated model finds to fit (highly persistent) inflation well is to attribute a disproportionate role to nominal rigidities. Then, in this world, deviations of inflation from target are persistent and represent persistent deviations of the economy from its steady state. Hence, the forecastability of other variables is also high.[11]

Our analysis seems thus to support the views in Cogley and Sbordone (2008) or Cogley et al. (2010) that the central bank target is time-varying (call it trend inflation), even after Volcker's disinflation. According to Cogley and Sbordone (2008), once movements in trend inflation are taken into account, the indexation component of a general New Keynesian Phillips curve is not needed to fit the data well. Notice that this interpretation does not downplay nominal rigidities. It suggests instead possible compatibility between a *modified* NK model and the forecast accuracy obtained by professional forecasters. Needless to say, this modified model is not the version NK—only W rigid., because this model retains a stationary inflation rate. In any case, it does not exacerbate the likely misspecification of the process for inflation. To verify this conjecture on the role of nonstationary inflation in delivering these results, we will introduce a varying inflation target in the SW07 model.

Now, in order to trust this reasoning, we should be able to rule out other explanations for the discrepancy between model and professional forecasts. Suppose for instance that professional forecasters are somehow biased or that their forecast errors are often large because of herding behavior or mood. In this case we may find little (or poor) forecastability just because professional forecasters do not use information efficiently. Then, if we take their forecasts too seriously, we might end up concluding (wrongly) that theoretical models should display little forecastability. Now, the fact is that there is strong evidence on the superiority (or, in the worst cases, noninferiority) of SPF forecasts vis-à-vis several alternative methods. We resort again to results in Ang et al. (2007) and Faust and Wright (2012) and the analyses in Rubaszek and Skrzypczynski (2008), Kolasa et al. (2012), Baghestani (2009, 2012), Stark (2010), or Valle e Azevedo and Pereira (2013).

Moving now to the model side, suppose that some of the parameters are time-varying, or that there are additional shocks affecting equilibrium conditions including in such a way that the model can be rewritten as one with time-varying parameters. If the shocks are not persistent, assuming fixed parameters instead of the correct specification may lead to overestimation of the forecastability. The same can occur if agents must learn the parameters of the model but we assume fixed parameters instead. Other common assumptions, such as the representative agent assumption, might be unreasonable; suppose instead that heterogeneous agents operate in the economy with different (smaller) information sets. Then assuming a representative agent who observes all the shocks might lead to overestimating the forecastability in the model. More generally, most modeling assumptions related to the number, persistence, and volatility of shocks, market structures, information structures, etc., have an impact on forecastability. There are certainly many potential explanations for the divergence between the strong predictability found in the model and that in the data (professional forecasters). We certainly admit the difficulties in ruling out these alternative explanations. Put another way, our results are model-dependent.

Finally, we would like to stress that our measure of closeness between model and (forecast) data seems to give different insights relative to a standard assessment of fit; notice that changes in the model, such as removing price rigidities, give rise to a better fit with forecast data although the estimated (fitted) model had price rigidities, i.e., the associated parameters were not estimated to be close to zero. Further, if the model is estimated assuming a stationary inflation rate when instead inflation has a unit root, it can offer a good match in terms of second moments, but it is hopelessly misspecified. Comparison of moments will not reveal the misspecification, whereas comparisons of forecastability measures may reveal it, as we have argued.

3.4. The Model with a Varying Inflation Target

We introduce a varying inflation target in the Smets and Wouters model, following Juillard et al. (2008) very closely. First, the (gross) inflation target, $\overline{\Pi}_t$, is assumed to evolve according to $\overline{\Pi}_t = \overline{\Pi}_{t-1} \exp(\varepsilon_t^{\pi^*})$ or, in log-linearized form, $\overline{\pi}_t = \overline{\pi}_{t-1} + \varepsilon_t^{\overline{\pi}}$, where the lower cases denote log deviations from the steady state and $\varepsilon_t^{\overline{\pi}}$ is a white-noise shock to the inflation target, assumed to be uncorrelated with the other shocks in the model. Second, the Taylor rule is adapted to accommodate the varying inflation target, taking the following form:

$$R_t = R_{t-1}^{\rho} \left[\left(\frac{\Pi_t}{\overline{\Pi}_t} \right)^{r_\pi} \frac{\gamma}{\beta} \overline{\Pi}_t \left(\frac{Y_t}{Y_t^*} \right)^{r_y} \left(\frac{Y_t}{Y_t^*} / \frac{Y_{t-1}}{Y_{t-1}^*} \right)^{r_{\triangle y}} \right]^{1-\rho} \exp(\varepsilon_t^r),$$

where R_t is the gross nominal interest rate, Π_t is gross inflation, Y_t is output, Y_t^* is potential output (i.e., output under flexible prices), and ε_t^r is a monetary policy shock. β is the discount factor of households, γ is the growth rate of the economy in the steady state, and ρ, r_π, r_y, and $r_{\triangle y}$ govern the reactions of the monetary authority. In order to solve the model, R_t and Π_t are stationarized as $R_t/\overline{\Pi}_t$ and $\Pi_t/\overline{\Pi}_t$, which justifies the additional term $\overline{\Pi}_t$ in the Taylor rule. In log-linearized form this reads as $r_t = \rho(r_{t-1} - \varepsilon_t^{\overline{\pi}}) + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^*)) + r_{\triangle y}((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)) + \varepsilon_t^r$, where r_t, π_t are log deviations of $R_t/\overline{\Pi}_t$ and $\Pi_t/\overline{\Pi}_t$ from their respective steady states and the remaining lower cases denote log deviations from the steady state.

We keep following Juillard et al. (2008) in assuming that whenever a firm is allowed to reoptimize its price (which occurs with probability $1-\xi_p$), it sets both the current price level, V_t , and the gross rate υ_t at which it will update its price from today until the time it is next allowed to change its policy. If at time $t+k$ the firm keeps its time-t policy (i.e., it was not allowed to change policy from t to $t+k$), its price is therefore $P_{t+k}(j) = V_t(j)(\upsilon_t(j))^k$. As emphasized by Juillard et al. (2008), this way of modeling the price setting, by letting firms choose two instead of one pricing variable optimally, imposes fewer exogenous constraints on the firm's profit maximization problem than a model with indexation. In this important sense the model is therefore less ad hoc. The profit maximization problem of a generic firm is then

$$\underset{V_t, \upsilon_t}{\text{Max}} E_t \left[\sum_{k=0}^{\infty} (\xi_p \beta)^k \lambda_{t+k} \left(\left[\frac{V_t (\upsilon_t)^k}{P_{t+k}} \right]^{1-\sigma_t} Y_{t+k} - \text{mc}_{t+k} \left\{ \left[\frac{V_t (\upsilon_t)^k}{P_{t+k}} \right]^{-\sigma_t} \right\} \right) \right],$$

where $E_t[.]$ is the expectation at time t, λ_{t+k} is the marginal utility of consumption, Y_{t+k} is aggregate output, mc_{t+k} is the firm's marginal cost, and σ_t is the elasticity of substitution across varieties of goods. Household nominal wage setting is modeled in an equivalent way [see Juillard et al. (2008) for all the details]. All this results in the price and wage Phillips curves being replaced (each) by a set of three equations (all these can be found in Appendix C, along with additional auxiliary equations). Further, we need to modify the measurement equations associated

TABLE 4. Theoretical forecast models (versions of SW07($\overline{\pi}_t$))

Model designation	Price rigidities	Wage rigidities	Observables
RBC	No	No	C, I, Y, W, H
NK($\overline{\pi}_t$)—only W rigid.	No	Yes	C, I, Y, W, R, Π, H
NK($\overline{\pi}_t$)	Yes	Yes	C, I, Y, W, R, Π, H

Notes: *C*, consumption; *I*, investment; *Y*, output; *W*, wages (all in log differences); *H*, hours (in levels); *R*, nominal Int. rate; Π, inflation (in log differences).

with inflation and the nominal interest rate. Because these two variables are now nonstationary and normalized in the model as $R_t/\overline{\Pi}_t$ and $\Pi_t/\overline{\Pi}_t$ we use instead $\log(R_t) - \log(R_{t-1}) = r_t - r_{t-1} + \varepsilon_t^{\overline{\pi}}$ and $\log(\Pi_t) - \log(\Pi_{t-1}) = \pi_t - \pi_{t-1} + \varepsilon_t^{\overline{\pi}}$; please refer to Appendix C for a detailed explanation. This means that in analyzing the theoretical forecastability we must look at the growth of inflation and at the growth of nominal interest rates, not at the levels.

We have estimated this alternative model using the sample employed in SW07 (1984q1–2004q4) while following all their settings very closely. The priors for the parameters we estimate are those used in SW07 (except for hours, where we employ a looser prior). There are two main differences nonetheless: we fix β at SW07's estimated value (0.998), as it was troublesome to estimate it; also, we use log utility to simplify the normalization of the model, because of the inclusion of the varying inflation targets. We should highlight the overall stability of the mode of the posterior distribution vis-à-vis the original SW07 model (see Appendix D for all the details). However, we naturally observe relevant departures in the parameters related to nominal rigidities and to the processes followed by the price and wage markup shocks. This would be expected given the different formulations of price and wage rigidities.

Next, we take this model (with the parameters set to the mode of the posterior distribution) and analyze, exactly as in Section 3.2, its theoretical forecastability.[12] We look also at the RBC version of this model (no restrictions on price and wage setting) and a version with only wage rigidities. These versions keep fixed, as before, the remaining structural parameters. Table 4 summarizes the specifications under scrutiny and Table 5 looks, as before, at the model's theoretical RMSFEs (normalized by the standard deviation of the variables) at various horizons. We compare this with the normalized (by the standard deviation of the observed variables) RMSFEs obtained by the SPF over the period 1984q1–2009q2. We repeat the measures of fit previously reported for the original SW07 model. Notice, however, that because interest rates and inflation are now in growth rates, we cannot compare the fit of the two main types of model (i.e., with fixed or varying inflation target) including these (different) variables. But we can and do compare the fits for the common variables C, I, Y, H. Several conclusions follow:

— The predictability of most variables is lower vis-à-vis the original SW07 model and much closer to that of professional forecasters. The measure of fit for the set

TABLE 5. Theoretical root-mean-square forecast error of models vs. SPF

Variable	Model	Horizon								
		1	2	3	4	5	6	8	12	20
Real GDP growth	SPF	0.76	0.88	0.97	0.99	1.02	—	—	—	—
	RBC	0.96	0.97	**0.98**	**0.99**	**0.99**	0.99	1.00	1.00	1.00
	NK($\overline{\pi}_t$)—only W rigid.	0.82	0.91	0.95	0.97	0.98	0.98	0.99	0.99	1.00
	NK($\overline{\pi}_t$)	**0.79**	**0.89**	0.94	0.95	0.96	0.97	0.97	0.98	0.98
Consumption growth	SPF	0.80	0.89	0.97	1.01	1.00	—	—	—	—
	RBC	0.78	0.88	0.90	0.90	0.90	0.91	0.93	0.98	1.00
	NK($\overline{\pi}_t$)—only W rigid.	**0.78**	0.91	**0.96**	**0.98**	**0.98**	0.98	0.99	0.99	1.00
	NK($\overline{\pi}_t$)	0.74	**0.89**	0.94	0.96	0.97	0.97	0.97	0.98	0.98
Invest. nonresid. growth	SPF	0.67	0.80	0.89	0.93	0.98	—	—	—	—
	RBC	0.80	0.90	0.92	**0.92**	**0.93**	0.93	0.95	0.98	1.00
	NK($\overline{\pi}_t$)—only W rigid.	0.72	0.84	0.88	0.90	0.90	0.92	0.94	0.97	0.99
	NK($\overline{\pi}_t$)	**0.72**	**0.84**	**0.88**	0.90	0.90	0.92	0.94	0.97	1.00
Hours (exc. SPF) level	SPF (Unemployment)	0.13	0.29	0.45	0.62	0.74	—	—	—	—
	RBC	0.48	0.63	0.72	**0.78**	**0.82**	0.84	0.87	0.89	0.92
	NK($\overline{\pi}_t$)—only W rigid.	**0.12**	**0.20**	**0.27**	0.34	0.40	0.46	0.56	0.71	0.88
	NK($\overline{\pi}_t$)	0.11	0.18	0.25	0.31	0.37	0.43	0.52	0.66	0.81
Wages growth	SPF	—	—	—	—	—	—	—	—	—
	RBC	0.98	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00
	NK($\overline{\pi}_t$)—only W rigid.	0.93	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00
	NK($\overline{\pi}_t$)	0.93	0.96	0.97	0.98	0.98	0.98	0.99	0.99	1.00
Inflation growth	SPF	0.94	1.06	1.00	0.98	1.00	—	—	—	—
	NK($\overline{\pi}_t$)—only W rigid.	0.62	**0.99**	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00
	NK($\overline{\pi}_t$)	**0.90**	0.98	0.99	**1.00**	**1.00**	1.00	1.00	1.00	1.00
	NK($\overline{\pi}_t$) - Univariate	0.98	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 5. Continued

		Horizon								
Variable	Model	1	2	3	4	5	6	8	12	20
Int. Rate	SPF	0.32	0.97	1.00	1.09	1.09	—	—	—	—
growth	NK($\overline{\pi}_t$)—only W rigid.	**0.74**	**0.86**	**0.91**	**0.94**	**0.96**	0.97	0.98	0.99	0.99
	NK($\overline{\pi}_t$)	0.74	0.85	0.91	0.94	0.95	0.96	0.97	0.98	0.98
Fit	RBC			—						
C, I, Y, R, Π, H	NK($\overline{\pi}_t$)—only W rigid.			**0.045**						
	NK($\overline{\pi}_t$)			0.048						
Fit	RBC			0.060						
C, I, Y, H	NK($\overline{\pi}_t$)—only W rigid.			**0.038**						
	NK($\overline{\pi}_t$)			0.044						
Fit	RBC			0.058						
C, I, Y, H	NK—only W rigid.			0.051						
From Table 3	NK—W rigid. + W index.			0.063						
	NK—no P index.			0.068						
	NK			0.075						

Notes: Forecastability of theoretical models vs. SPF. Normalized (or relative to the standard deviation of the variables) RMSFE at different forecast horizons. The standard deviation is that of the model variables in the case of model forecasts or that in the data in the case of SPF forecasts. SPF's evaluation period is 1984q1–2009q2. Bold entries represent, for each horizon and variable, the values closest to the SPF. *Fit* refers to the geometric mean of the differences between the normalized RMSFEs obtained with the SPF and those obtained with the model, with the horizons from $h = 1$ to $h = 5$ and the variables below Fit pooled together.

of comparable variables (C, I, Y, H) is much lower for the model with a varying inflation target, compared to the various versions of SW07 analyzed earlier in Table 3. The version with only wage rigidities has the best fit even if, in the case of hours, the discrepancy with professional forecasts is higher than that obtained in the original version of the model.

— Inflation and nominal interest rates (which are in growth rates) are naturally much less predictable now, whereas we observe some discrepancy at $h = 1$ between professional and model forecasts. We recall that in the case of nominal interest rates (in which case the data are released without delays), professional forecasters have an informational advantage of about half a quarter (see Appendix A for details). The implied differences in forecastability are thus overestimated.

In short, the forecastability of this modified version of the SW07 model is more in line with that of SPF forecasts.

We have complemented this analysis by asking whether a better fit in terms of theoretical forecastability was associated with better empirical (with actual data) forecast performance.[13] We have considered all the versions of the SW07 model analyzed thus far, including versions with and without a varying inflation target. In short, even though we found an empirical deterioration of the forecast performance (i.e., higher relative RMSFEs, which would be expected due, e.g., to parameter estimation), there is nonetheless a resemblance between the ranking of the models in terms of empirical forecast performance and that in the theoretical analysis. Importantly, the consideration of the model with a varying inflation target resulted in several instances in enhanced empirical performance, except at long horizons.

4. CONCLUDING REMARKS

It seems unwise to expect too much from macroeconomic forecasts. For what really matters (real variables, except for unemployment), best practice has little to say at horizons greater than three or four quarters. If these facts inform general equilibrium modeling, they probably say the model economy should be characterized by low forecastability (again, except for unemployment—hours). This does not occur with the standard New Keynesian model analyzed here, whereas the flexible-prices (or RBC) version goes much too far, especially given the behavior of hours.

Our analysis suggests that care should be taken at least in the way trend inflation, or varying central bank target, is modeled. In the original NK model analyzed here and many others, the central bank target, which is steady-state inflation, is fixed, implying that any deviation of inflation from target is necessarily interpreted as a deviation from the steady state (inflation gap). Once the model is fitted to the data, the likely misspecification shows up in the form of significant nominal rigidities, which in turn imply a high theoretical forecastability of inflation and other variables, as we have shown. This is clearly at odds with the data (professional forecasts). With a time-varying inflation target, the forecastability of most variables decreases and becomes more compatible with the fact that professional forecasters have a hard time.

Finally, the full flexibility of prices and (especially) wages implies too low a forecastability of hours, at odds with the data.

NOTES

1. Notice, however, that, using data up to 1991, Romer and Romer (2000) have shown that Green-Book's forecasts of inflation and real GDP are statistically unbiased and dominate private sector forecasts (suggesting that the Federal Reserve has considerably more information than the private sector). The period of the "Great Moderation" between 1982 and 2007 [see, e.g., McConnell and Perez-Quiros (2000), Stock and Watson (2003), and Giannone et al. (2008)] has affected the time-series properties of many variables as well as the forecast performance of different models and surveys. In particular, D'Agostino and Whelan (2008) show that the superior forecasting performance of the Green-Book forecasts deteriorated considerably after 1991. Similarly, Gamber and Smith (2009) find that the Fed's relative forecasting superiority has declined relative to SPF forecasts for both inflation and real GDP growth after 1994, consistent with evidence in Gavin and Mandal (2003). We have reached similar conclusions within a sample ending in 2003q4.

2. For complete information on the survey's background see http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf, as well as Zarnowitz (1969), Croushore (1993), and Zarnowitz and Braun (1993).

3. The individual respondents' point forecasts are generally close to the central tendencies of their subjective distributions [see, e.g., Engelberg et al. (2009)], whereas there is clear evidence that this aggregation produces forecasts that are in general superior to individual forecasts. Obviously, a not-so-straightforward aggregation can result in forecast improvements, and this can be achieved even when there is (as in SPF) entry and exit of forecasters; see Capistrán and Timmerman (2009).

4. We should refer that an analysis of subsamples within the post-1984 sample, certainly available upon request, would not change the main conclusions reported here. For brevity we have decided to omit such an analysis.

5. We use the August 2010 vintage of data. We should refer that using real-time vintages would not change the conclusions of this section qualitatively. We chose to use the latest vintage of data in order to be consistent with the data used in the DSGE model in Section 3. For a discussion of real-time data see, e.g., Croushore and Stark (2001, 2003).

6. In practice we include in $\mathbf{s}_t$ a constant and the lags of the variables output, investment, consumption, and wages, because for these variables we focus on the forecastability of the log differences. For example, the measurement equation for consumption is $\log(C_t^{\text{obs}}) - \log(C_{t-1}^{\text{obs}}) = c_t - c_{t-1} + \gamma$, where lower case denotes log deviations from the steady-state, γ is the growth rate of the economy in the steady-state, and the superscript obs indicates that the variable is observed. In the case of inflation (π_t), the measurement equation is simply $\pi_t^{\text{obs}} = \overline{\pi} + \pi_t$, where $\overline{\pi}$ is steady-state inflation and π_t represents log deviation of (gross) inflation from its steady state.

7. Thus, we are implicitly analyzing the forecastability of the model relative to a long-run forecast or a relative forecastability measure; see Granger and Newbold (1986) or Diebold and Kilian (2001). This is so because we are implicitly picking as benchmark forecast the constant that delivers the lowest RMSFE (and the RMSFE is equal to the standard deviation). Because all the (transformed) variables are assumed to be stationary, the optimal mean square long-run forecast is this constant. Obviously, unlike the real-time average analyzed in Section 2, this constant forecast is unfeasible in real time.

8. The measure of fit is hence $(\Pi_i \Pi_h |\text{RMSFE}_{i,h}(\text{SPF}) - \text{RMSFE}_{i,h}(\text{Model})|)^{1/(\#\text{Variables}\#\text{horizons})})$, $i = C, I, Y, R, \Pi, H$ (or a subset), $h = 1, 2, 3, 4, 5$), where $\text{RMSFE}_{i,h}(\text{SPF})$ is the normalized RMSFE obtained with the SPF for variable i at horizon h and $\text{RMSFE}_{i,h}(\text{Model})$ is that obtained with the model. The lower this measure, the better the fit.

9. It is important to note that this feature of the specific New Keynesian model analyzed here is certainly common to any model featuring price and wage setting frictions along with an important indexation mechanism (to target or current inflation or a combination of the two) aimed at rationalizing the observed persistence of inflation; see, e.g., the models in Christiano et al. (2005), Schorfheide (2005), Adolfson et al. (2007a, 2007b), or Ireland (2007). This occurs because nominal rigidities generate high persistence in inflation and in other variables (and thus strong forecastability).

10. They are out of bounds in the case of the nominal interest rate when $h = 1$. But in this particular case we note the informational advantage of SPF panelists. When $h = 1$ they already know the relevant nominal interest rate data for half of the quarter to which the forecast refers. This informational advantage actually occurs with all the variables, although to a lesser extent, because they are typically released with a delay of half a quarter (although other relevant information is surely available in the middle of the quarter, when SPF forecasts are constructed). Thus, for the nominal interest rate, the relative RMSFE obtained with SPF is surely *deflated* relative to what would occur if the comparison with the models were totally fair. For the other variables under scrutiny this may also occur, but to a lesser extent. Hence, and especially in the case of the nominal interest rate, models with *fewer* rigidities may be more in line with the SPF.

11. Similarly, the behavior of the theoretical forecasts of hours in the RBC model seems at odds with the data. We recall that in the case of the SPF the forecasts are not of hours, but of unemployment; hence we avoid stretching any argument too far. Still, introducing only wage rigidities is enough to get closer to the data while resulting in much improved empirical forecasts, as we will later remark.

12. We should note that in this extension we are introducing another shock (the inflation target shock $\varepsilon_t^{\overline{\pi}}$) while keeping the number of observables used to compute the theoretical forecasts. That is, we are computing $E[\mathbf{y}_{T+h}|\mathbf{y}_{T,}\mathbf{y}_{T-1},\ldots,\mathbf{y}_1]$ instead of $E[\mathbf{y}_{T+h}|\varepsilon_T,\varepsilon_{T-1},\ldots,\varepsilon_1]$, which is what an agent in the economy would be able to compute. We have verified that including another observable from the state vector in the observables $\mathbf{y}_{T,}\mathbf{y}_{T-1},\ldots,\mathbf{y}_1$ (which would result in obtaining $E[\mathbf{y}_{T+h}|\varepsilon_T,\varepsilon_{T-1},\ldots,\varepsilon_1]$) would result in minimal differences in the results.

13. All these results are available upon request. We focused on the sample 1984q1–2009q2 for evaluation purposes, whereas the parameterizations are exactly those employed in the analysis of the theoretical predictability.

14. http://www.phil.frb.org/econ/spf/spfpage.html. For a recent discussion about the SPF see Croushore (2006).

15. For example, to forecast quarterly CPI inflation for 2014Q2 in the middle of 2014Q1, SPF panelists know the CPI figures until January 2014 and other information (such as oil prices) until mid-February. We denote this forecast as a two quarter ahead forecast (i.e., $h = 2$) but compare it with forecasts (naive and model-based) constructed with information referring only to the end of December of 2013 and before, although sometimes only available by mid-February. In the case of series released with national accounts, and because of release delays, the latest figures of these series known by SPF panelists coincide approximately with those contained in the information sets we define. For example, the initial release of real output growth in mid-February of 2014 refers to the fourth quarter of 2013 and it is contained in the information sets we build for the forecast moment "2013:Q4." Still, SPF panelists surely make use of other information released until the middle of the quarter. Again, we use only information referring to the previous quarter and before.

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APPENDIX A: DATA

Our data for the SPF predictions come directly from the Federal Reserve Bank of Philadelphia Web site and cover the period 1984q1–2009q2;[14] The August 2010 vintage of all series (against which forecasts are compared) was downloaded from the FRED database (Federal Reserve Bank of St. Louis). Data are converted to quarterly whenever the series are available at a higher frequency (by averaging the observations within each quarter). Except for unemployment and interest rates, all data is in growth rates. Except for interest rates, all published data are seasonally adjusted, in accordance with the targets of professional forecasters. Prior to 1992, nominal and real output forecasts refer to nominal GNP. GDP deflator forecasts refer to GNP deflator prior to 1992, to GDP deflator from 1992 through 1995, and to chain-weighted price index for GDP since 1996.

Table A.1 shows the definition of all series, SPF's and FRED's ID codes.

As for data used in SW07, we note that the August 2010 vintage of the following series is used: FRED's GDPC1 (real output), PCECC96 (consumption), and PNFIC96 (investment), which are divided by the Bureau of Labor Statistics (BLS) series LNU00000000Q (civilian noninstitutional population, 16 years and over). The short-term nominal interest rate is the Federal funds rate (FRED's FEDFUNDS); inflation is measured with FRED's GDPDEF (perfect match with SPF); and hours worked is obtained as average weekly hours (nonfarm business, the BLS's PRS8500602) multiplied by civilian employment (16 and over, FRED's CE16OV) and then divided by the BLS's series LNU00000000Q (civilian noninstitutional population, 16 years and over).

TABLE A.1. Definitions of data series

Definition	FRED code	SPF code
Gross Domestic Product (nominal)	GDP	NGDP
Gross Domestic Product (real)	GDPC1	RGDP
Real personal consumption expenditures	PCECC96	RCONSUM
Real private nonresidential fixed investment	PNFIC96	RNRESIN
Real private residential fixed investment	PRFIC96	RRESINV
Housing starts total: new privately owned housing units	HOUST	HOUSING
Industrial production index	INDPRO	INDPROD
Real federal cons. exp. & gross investment	FGCEC1	RFEDGOV
Real state & local cons. exp. & gross investment	SLCEC1	RSLGOV
Civilian unemployment rate	UNRATE	UNEMP
Consumer Price Index: all items	CPIAUCSL	CPI
Gross Domestic Product deflator	GDPDEF	PGDP
10-year Treasury constant maturity rate	GS10	TBOND
3-month Treasury bill: secondary market rate	TB3MS	TBILL

In Section 2 we simulate a real-time situation to construct forecasts while making sure there is no advantage over SPF forecasts in terms of timing. More precisely, in order to forecast y_{t+h} at quarter t, the information set we consider contains data referring only to quarter t and earlier, including data that become available only around the middle of quarter $t+1$ (e.g., national accounts data) but before SPF panelists submit what we denote as h quarters ahead forecasts of y_t (i.e., forecasts of y_{t+h}). To be clear, SPF participants report forecasts for what we denote as quarter $t+h$, $h = 1, 2, 3, 4$, in the middle of quarter $t+1$. This means that, especially in the case of forecasts of CPI inflation or 3-month T-bill rates, there is an informational advantage of the survey participants relative to the naive or model based forecasts.[15] In any case, this informational advantage is only a concern for very short horizons.

APPENDIX B: SMETS AND WOUTERS (2007) MODEL

Here we present the log-linearized equations of the SW07 model (sticky-price–wage economy) and also of the flexible prices version (variables denoted with an asterisk); see Table B.1. The model variables are (in log deviations from the steady state) output (y_t), consumption (c_t), investment (i_t), utilized and installed capital ($k_t^s,\ k_t$), capacity utilization (z_t), rental rate of capital (r_t^k), Tobin's q (q_t), price and wage markup ($\mu_t^p,\ \mu_t^w$), inflation rate(π_t), real wage (w_t), total hours worked (l_t), and nominal interest rate (r_t). The shocks are total factor productivity (ε_t^a), investment-specific technology (ε_t^i), government purchases (ε_t^g), risk premium (ε_t^b), monetary policy (ε_t^r), wage markup (ε_t^w), and price markup (ε_t^p). The equations are

(1) $y_t = c_y c_t + i_y i_t + r^{kss} k_y z_t + \varepsilon_t^g$,
(2) $c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma} \mathrm{E}_t c_{t+1} + \frac{w^{ss} l^{ss} (\sigma_c - 1)}{c^{ss} \sigma_c (1+\lambda/\gamma)} (l_t - \mathrm{E}_t l_{t+1})$
$- \frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c} (r_t - \mathrm{E}_t \pi_{t+1}) - \frac{1-\lambda/\gamma}{(1+\lambda\lambda/\gamma)\sigma_c} \varepsilon_t^b$,
(3) $i_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t i_{t+1} + \frac{1}{\varphi\gamma^2 (1+\beta\gamma^{(1-\sigma_c)})} q_t + \varepsilon_t^i$,
(4) $q_t = \beta(1-\delta)\gamma^{-\sigma_c} \mathrm{E}_t q_{t+1} - r_t + \mathrm{E}_t \pi_{t+1} + (1 - \beta(1-\delta)\gamma^{-\sigma_c}) \mathrm{E}_t r_{t+1}^k - \varepsilon_t^b$,
(5) $y_t = \phi_p (\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a)$,
(6) $k_t^s = k_{t-1} + z_t$,
(7) $z_t = \frac{1-\psi}{\psi} r_t^k$,
(8) $k_t = (1-\delta)/\gamma k_{t-1} + (1 - (1-\delta)/\gamma) i_t + (1 - (1-\delta)/\gamma)\varphi\gamma^2 (1 + \beta\gamma^{(1-\sigma_c)}) \varepsilon_t^i$,
(9) $\mu_t^p = \alpha(k_t^s - l_t) - w_t + \varepsilon_t^a$,
(10) $\pi_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p \beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t \pi_{t+1} + \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p} \pi_{t-1} - \frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_p)(1-\xi_p)}{(1+\iota_p\beta\gamma^{(1-\sigma_c)})(1+(\phi_p - 1)\varepsilon_p)\xi_p} \mu_t^p + \varepsilon_t^p$,
(11) $r_t^k = l_t + w_t - k_t$,
(12) $\mu_t^w = w_t - \sigma_l l_t - \frac{1}{1-\lambda/\gamma} (c_t - \lambda/\gamma c_{t-1})$,
(13) $w_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} (\mathrm{E}_t w_{t+1} + \mathrm{E}_t \pi_{t+1}) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} (w_{t-1} + \iota_w \pi_{t-1}) - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}} \pi_t$,
$- \frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_w)(1-\xi_w)}{(1+\beta\gamma^{(1-\sigma_c)})(1+(\phi_w - 1)\varepsilon_w)\xi_w} \mu_t^w + \varepsilon_t^w$
(14) $r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y (y_t - y_t^*)) + r_{\triangle y}((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)) + \varepsilon_t^r$,
(15) $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$,
(16) $\varepsilon_t^b = \rho_a \varepsilon_{t-1}^b + \eta_t^b$,
(17) $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^a + \rho_{ga} \eta_t^a + \eta_t^g$,
(18) $\varepsilon_t^i = \rho_I \varepsilon_{t-1}^I + \eta_t^I$,
(19) $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$,
(20) $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$,
(21) $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$.

The asterisk-model (flexible prices) variables are (in log deviations from the steady state) output (y_t^*), consumption (c_t^*), investment (i_t^*), utilized and installed capital ($k_t^{s*},\ k_t^*$), capacity utilization (z_t^*), rental rate of capital (r_t^{k*}), Tobin's q (q_t^*), price and wage markup ($\mu_t^{p*},\ \mu_t^{w*}$), real wage (w_t^*), and total hours worked (l_t^*). The equations are

(1∗) $y_t^* = c_y c_t^* + i_y i_t^* + r^{kss} k_y z_t^* + \varepsilon_t^g$,
(2∗) $c_t^* = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1}^* + \frac{1}{1+\lambda/\gamma} \mathrm{E}_t c_{t+1}^* + \frac{w^{ss} l^{ss} (\sigma_c - 1)}{c^{ss} \sigma_c (1+\lambda/\gamma)} (l_t^* - \mathrm{E}_t l_{t+1}^*)$
$- \frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c} r_t^* - \frac{1-\lambda/\gamma}{(1+\lambda\lambda/\gamma)\sigma_c} \varepsilon_t^b$,
(3∗) $i_t^* = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1}^* + \frac{\beta\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t i_{t+1}^* + \frac{1}{\varphi\gamma^2 (1+\beta\gamma^{(1-\sigma_c)})} q_t^* + \varepsilon_t^i$,
(4∗) $q_t^* = \beta(1-\delta)\gamma^{-\sigma_c} \mathrm{E}_t q_{t+1}^* - r_t^* + (1 - \beta(1-\delta)\gamma^{-\sigma_c}) \mathrm{E}_t r_{t+1}^{k*} - \varepsilon_t^b$,
(5∗) $y_t^* = \phi_p (\alpha k_t^{s*} + (1-\alpha) l_t^* + \varepsilon_t^a)$,
(6∗) $k_t^{s*} = k_{t-1}^* + z_t^*$,
(7∗) $z_t^* = \frac{1-\psi}{\psi} r_t^{k*}$,
(8∗) $k_t^* = (1-\delta)/\gamma k_{t-1}^* + (1 - (1-\delta)/\gamma) i_t^* + (1 - (1-\delta)/\gamma)\varphi\gamma^2 (1 + \beta\gamma^{(1-\sigma_c)}) \varepsilon_t^i$,
(9∗) $\mu_t^{p*} = \alpha(k_t^{s*} - l_t^*) - w_t^* + \varepsilon_t^a$,
(10∗) $\mu_t^{p*} = 1$,
(11∗) $r_t^{k*} = l_t^* + w_t^* - k_t^*$,
(12∗) $\mu_t^{w*} = -\sigma_l l_t^* - \frac{1}{1-\lambda/\gamma} (c_t^* + \lambda/\gamma c_{t-1}^*)$,
(13∗) $w_t^* = \mu_t^{w*}$.

TABLE B.1. Priors and posterior (taken from SW07)

Parameter	Interpretation	Prior			Posterior
		Density	Mean	Std.	mode
φ	Invest. adj. cost	$\mathcal{N}$	4.000	1.500	6.23
σ_c	Inv. elast. intert. subst.	$\mathcal{N}$	1.500	0.375	1.47
λ	Habit	$\mathcal{B}$	0.700	0.100	0.68
ξ_w	Wage rigidity	$\mathcal{B}$	0.500	0.100	0.74
σ_l	Inv. elast. hours	$\mathcal{N}$	2.000	0.750	2.30
ξ_p	Price rigidity	$\mathcal{B}$	0.500	0.100	0.73
ι_w	Wage indexation	$\mathcal{B}$	0.500	0.150	0.46
ι_p	Price indexation	$\mathcal{B}$	0.500	0.150	0.21
ψ	Cap. utilization cost	$\mathcal{B}$	0.500	0.150	0.69
Φ	Fixed cost	$\mathcal{N}$	1.250	0.125	1.54
r_π	Response to inflation	$\mathcal{N}$	1.500	0.250	1.77
ρ	Int. rate smoothing	$\mathcal{B}$	0.750	0.100	0.84
r_y	Response to output	$\mathcal{N}$	0.125	0.050	0.08
$r_{\triangle y}$	Response to output growth	$\mathcal{N}$	0.125	0.050	0.16
$\bar{\pi}$	Steady state infl.	$\mathcal{G}$	0.625	0.100	0.67
β'	Discount factor*	$\mathcal{G}$	0.250	0.100	0.12
$\bar{l}$	Steady state hours	$\mathcal{N}$	0.000	2.000	−0.55
γ	Trend growth rate	$\mathcal{N}$	0.400	0.100	0.44
α	Capital share	$\mathcal{N}$	0.300	0.050	0.21
δ	Depreciation rate†	—	—	—	0.025
μ^w_{SS}	Wage markup†	—	—	—	1.50
g_y	Government/output†	—	—	—	0.18
ρ_a	AR prod. shock	$\mathcal{B}$	0.500	0.200	0.94
ρ_b	AR risk premium	$\mathcal{B}$	0.500	0.200	0.14
ρ_g	AR government	$\mathcal{B}$	0.500	0.200	0.96
ρ_I	AR investment	$\mathcal{B}$	0.500	0.200	0.64
ρ_r	AR mon. policy	$\mathcal{B}$	0.500	0.200	0.29
ρ_p	AR price markup	$\mathcal{B}$	0.500	0.200	0.74
ρ_w	AR wage markup	$\mathcal{B}$	0.500	0.200	0.82
μ_p	MA price markup	$\mathcal{B}$	0.500	0.200	0.59
μ_w	MA wage markup	$\mathcal{B}$	0.500	0.200	0.62
ρ_{ga}	Prod. shock in G	$\mathcal{B}$	0.500	0.200	0.39
σ_a	St.dev. prod. shock	$\mathcal{IG}$	0.100	2.000	0.35
σ_b	St.dev. risk premium	$\mathcal{IG}$	0.100	2.000	0.18
σ_g	St.dev. government	$\mathcal{IG}$	0.100	2.000	0.41
σ_I	St.dev. investment	$\mathcal{IG}$	0.100	2.000	0.39
σ_r	St.dev. mon. policy	$\mathcal{IG}$	0.100	2.000	0.12
σ_p	St.dev. price markup	$\mathcal{IG}$	0.100	2.000	0.11
σ_w	St.dev. wage markup	$\mathcal{IG}$	0.100	2.000	0.21
Estimation sample: 1984q1–2004q4					

Notes: $\mathcal{N}$ is normal distribution, $\mathcal{B}$ is beta -distribution, $\mathcal{G}$ is Gamma distribution, $\mathcal{IG}$ is inverse Gamma distribution.
* $\beta' = 100(\beta^{-1} - 1)$, where β is the discount factor.
† These parameters are assumed known in SW07.

APPENDIX C: SMETS AND WOUTERS (2007) MODEL WITH VARYING INFLATION TARGET

The model variables are output (y_t), consumption (c_t), investment (i_t), utilized and installed capital ($k_t^s,\ k_t$), capacity utilization (z_t), rental rate of capital (r_t^k), Tobin's q (q_t), price and wage markup ($\mu_t^p,\ \mu_t^w$), real wage (w_t), wage inflation (π_t^w), total hours worked (l_t), marginal rate of substitution between consumption and leisure (mrs_t), and marginal cost (mc_t), as well as the auxiliary variables ψ_t^p,υ_t^p, ψ_t^w, and υ_t^w. All these are in log deviations from the steady state. The nominal interest rate (r_t) and the inflation rate(π_t) are in log deviations from the target inflation rate. The shocks are total factor productivity (ε_t^a), investment-specific technology (ε_t^i), government purchases (ε_t^g), risk premium (ε_t^b), monetary policy (ε_t^r), wage markup (ε_t^w), price markup (ε_t^p), and inflation target ($\varepsilon_t^{\overline{\pi}}$).

The model, with the variables just described, comprises equations 1–8, 11, and 14–21 from the original SW07 model described in Appendix B (considering log utility or $\sigma_c = 1$) and equations 1*–13* of the asterisk economy. The Phillips curve (equation 10 in SW07) is replaced with the following three equations [taken from the Technical Appendix of Juillard et al. (2008)]:

$$(10)'\ \psi_t^p = \xi_p \psi_{t-1}^p + (1-\xi_p)\upsilon_{t-1}^p - \varepsilon_t^{\overline{\pi}},$$

$$(10)''\ E_t\upsilon_{t+1}^p = \upsilon_t^p + \frac{(1-\xi_p\beta)^2}{(\xi_p\beta)^2}\frac{\xi_p}{1-\xi_p}\psi_t^p - \frac{(1-\xi_p\beta)^2}{(\xi_p\beta)^2}\frac{\xi_p}{1-\xi_p}\pi_t + \frac{(1-\xi_p\beta)^2}{(\xi_p\beta)^2}(\text{mc}_t + \mu_t^p),$$

$$(10)'''\ E_t\pi_{t+1} = \pi_t\left(\frac{2}{\beta} - \xi_p\right) + (1-\xi_p)(1+\xi_p)\upsilon_t^p + \left((1-\xi_p)\xi_p - \frac{2}{\beta}\right)\psi_t^p - \frac{2(1-\xi_p)(1-\xi_p\beta)}{\xi_p\beta}(\text{mc}_t + \mu_t^p).$$

The wage Phillips curve (equation 13 in the original model) is replaced with the following three equations:

$$(13)'\ \psi_t^w = \xi_w \psi_{t-1}^w + (1-\xi_w)\upsilon_{t-1}^w - \varepsilon_t^{\pi^*},$$

$$(13)''\ E_t\upsilon_{t+1}^w = \upsilon_t^w + \frac{(1-\xi_w\beta)^2}{(\xi_w\beta)^2}\frac{\xi_w}{1-\xi_w}\psi_t^w - \frac{(1-\xi_w\beta)^2}{(\xi_w\beta)^2}\frac{\xi_p}{1-\xi_p}\pi_t + \frac{(1-\xi_w\beta)^2}{(\xi_w\beta)^2}(\text{mrs}_t - w_t + \mu_t^w),$$

$$(13)'''\ E_t\pi_{t+1}^w = \pi_t^w\left(\frac{2}{\beta} - \xi_w\right) + (1-\xi_w)(1+\xi_w)\upsilon_t^w + \left((1-\xi_w)\xi_w - \frac{2}{\beta}\right)\psi_t^w - \frac{2(1-\xi_w)(1-\xi_w\beta)}{\xi_w\beta}(\text{mrs}_t - w_t + \mu_t^w),$$

Other equations are the following:

$$w_t = w_{t-1} + \pi_t^w - \pi_t,$$

$$\text{mrs}_t = \sigma_l l_t - \sigma^w \sigma_l \frac{(1-\xi_w)}{\xi_w}\pi_t^w + \sigma^w \sigma_l \frac{(1-\xi_w)}{\xi_w}\psi_t^w - \frac{1}{1-\lambda/\gamma}(c_t - \lambda/\gamma c_{t-1}),$$

where $\sigma^w = \mu_{SS}^w/(\mu_{SS}^w - 1)$.

Also, the measurement equations of (gross) inflation and interest rates now involve first differences of the logs. This follows from the fact that inflation and interest rates are nonstationary in this model and must be normalized as $\Pi_t^* := \Pi_t/\overline{\Pi}_t$ and $R_t^* := R_t/\overline{\Pi}_t$, where $\overline{\Pi}_t$ is the inflation target. Then, because the model is log-linearized around its steady state, the solution to the model involves the variables r_t and π_t, which are *log deviations* of $R_t/\overline{\Pi}_t$ and $\Pi_t/\overline{\Pi}_t$ from their respective steady states. Now, neither $R_t/\overline{\Pi}_t$ nor $\Pi_t/\overline{\Pi}_t$ has a counterpart in observable data. However, notice that $\log(R_t) - \log(R_{t-1}) = \log(\overline{\Pi}_t R_t^*) - \log(\overline{\Pi}_{t-1} R_{t-1}^*) = \log(\overline{\Pi}_t) - \log(\overline{\Pi}_{t-1}) + \log(R_t^*) - \log(R_{t-1}^*) = \varepsilon_t^{\overline{\pi}} + r_t - r_{t-1}$ and, following a similar route, $\log(\Pi_t) - \log(\Pi_{t-1}) = \pi_t - \pi_{t-1} + \varepsilon_t^{\overline{\pi}}$. That is, if we take first differences of logs of R_t and Π_t, which can be computed in the data, we are able to map these variables to variables in the model.

APPENDIX D: SMETS AND WOUTERS (2007) MODEL WITH VARYING INFLATION TARGET—ESTIMATION RESULT

See Table D.1 for estimates of this model.

TABLE D.1. Priors and posterior: Model with varying inflation target

		Prior			Posterior	
Parameter	Interpretation	Density	Mean	Std.	Mode	Std.
φ	Invest. adj. cost	$\mathcal{N}$	4.000	1.500	5.08	1.20
σ_c	Inv. elast. intert. subst.	—	—	—	1.00	
λ	Habit	$\mathcal{B}$	0.700	0.100	0.73	0.03
ξ_w	Wage rigidity	$\mathcal{B}$	0.500	0.100	0.83	0.07
σ_l	Inv. elast. hours	$\mathcal{N}$	2.000	0.750	2.92	0.49
ξ_p	Price rigidity	$\mathcal{B}$	0.500	0.100	0.73	0.04
ψ	Cap. utilization cost	$\mathcal{B}$	0.500	0.150	0.83	0.07
Φ	Fixed cost	$\mathcal{N}$	1.250	0.125	1.44	0.08
r_π	Response to inflation	$\mathcal{N}$	1.500	0.250	1.25	0.41
ρ	Int. rate smoothing	$\mathcal{B}$	0.750	0.100	0.96	0.01
r_y	Response to output	$\mathcal{N}$	0.125	0.050	0.16	0.04
$r_{\triangle y}$	Response to output growth	$\mathcal{N}$	0.125	0.050	0.23	0.03
β	Discount factor	-	-	-	0.998	
$\bar{l}$	Steady state hours	$\mathcal{N}$	0.000	4.000	−1.47	2.39
γ	Trend growth rate	$\mathcal{N}$	0.400	0.100	0.44	0.04
α	Capital share	$\mathcal{N}$	0.300	0.050	0.22	0.04
δ	Depreciation rate†	—	—	—	0.025	
μ_{SS}^w	Wage markup†	—	—	—	1.50	
g_y	Government/output†	—	—	—	0.18	
ρ_a	AR prod. shock	$\mathcal{B}$	0.500	0.200	0.94	0.03
ρ_b	AR risk premium	$\mathcal{B}$	0.500	0.200	0.97	0.01
ρ_g	AR government	$\mathcal{B}$	0.500	0.200	0.94	0.02

TABLE D.1. Continued

		Prior			Posterior	
Parameter	Interpretation	Density	Mean	Std.	Mode	Std.
ρ_I	AR investment	$\mathcal{B}$	0.500	0.200	0.73	0.08
ρ_r	AR mon. policy	$\mathcal{B}$	0.500	0.200	0.37	0.09
ρ_p	AR price markup	$\mathcal{B}$	0.500	0.200	0.90	0.04
ρ_w	AR wage markup	$\mathcal{B}$	0.500	0.200	0.50	0.19
μ_p	MA price markup	$\mathcal{B}$	0.500	0.200	0.26	0.13
μ_w	MA wage markup	$\mathcal{B}$	0.500	0.200	0.49	0.21
ρ_{ga}	Prod. shock in G	$\mathcal{B}$	0.500	0.200	0.55	0.10
σ_a	St.dev. prod. shock	$\mathcal{IG}$	0.100	2.000	0.40	0.03
σ_b	St.dev. risk premium	$\mathcal{IG}$	0.100	2.000	0.04	0.01
σ_g	St.dev. government	$\mathcal{IG}$	0.100	2.000	0.45	0.03
σ_I	St.dev. investment	$\mathcal{IG}$	0.100	2.000	0.43	0.06
σ_r	St.dev. mon. policy	$\mathcal{IG}$	0.100	2.000	0.13	0.01
σ_p	St.dev. price markup	$\mathcal{IG}$	0.100	2.000	1.43	0.29
σ_w	St.dev. wage markup	$\mathcal{IG}$	0.100	2.000	41.9	10.7
Estimation sample: 1984q1–2004q4						

Notes: $\mathcal{N}$ is normal distribution, $\mathcal{B}$ is beta distribution, $\mathcal{G}$ is gamma distribution, $\mathcal{IG}$ is inverse gamma distribution.

† These parameters are fixed, as in SW07.